Performance Measure of Residual Vibration Control

Chul-Goo Kang
Professor
Department of Mechanical Engineering,
Konkuk University,
Hwayang-dong, Gwangjin-gu,
Seoul 143-701, Korea
e-mail: cgkang@konkuk.ac.kr

The robustness of residual vibration control, such as input shaping, has conventionally been evaluated from the ratio of residual vibration amplitude with input shaping to that without input shaping at the time of the final impulse. However, in that robustness evaluation, vibration-suppressing speed due to each residual vibration control has not been considered, which is also an important aspect of residual vibration control. In this paper, a performance measure including robustness to modeling errors and the effect of vibration-suppressing speed is defined, and the validity of the performance measure is demonstrated by simulations and experimental works. [DOI: 10.1115/1.4003377]

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1 Introduction

Input shaping, a method of residual vibration control, is a feedforward control technique for reducing vibrations in undamped or underdamped systems, which is implemented by convoluting a sequence of impulses with a desired command. The amplitudes and time locations of the impulses for input shaping are determined from the system’s natural frequencies and damping ratios by solving a set of constraint equations.

The early form of the input shaping called postcast control developed by Smith [1] in the late 1950s was motivated by a simple wave cancellation concept for the elimination of the oscillatory motion of the underdamped system and was unfortunately sensitive to modeling errors of natural frequency and damping ratio. Because of this sensitivity problem to modeling errors and lack of microprocessor technology at that time, the postcast control did not come into widespread use for real systems. However, an input shaping paper published in 1990 by Singer and Seering [2] renewed interest in prefiltering reference inputs for residual vibration reduction, which is a method that improved robustness to modeling errors by adding additional constraints on the derivative of residual vibration magnitudes. Given its robustness, input shaping has been implemented on a variety of systems including cranes [3], disk drives [4], flexible spacecrafts [5,6], and industrial robots [7,8] using microprocessor technology. Since Singer and Seering’s work [2] for linear second-order systems, the input shaping technique has progressed in such a way that is effective for multimode systems [9], for nonlinear systems [5,10], for time-varying systems [11], and for nonlinearities such as Coulomb friction [12], backlash [13], and on-off thrusters [14].

Robustness to modeling error is an important issue for practical usefulness of residual vibration control. The robustness of residual vibration control such as input shaping has conventionally been evaluated using sensitivity curves plotted from the ratio of residual vibration amplitude with input shaping to that without input shaping at the time of the final impulse [4,9,11,15–18]. However, this sensitivity curve does not represent the effect of vibration-suppressing speed due to each residual vibration control, which is also an important aspect of the residual vibration control.

In this paper, the robustness function and the performance measure including the effect of vibration-suppressing speed are defined. The validity of the aforementioned parameters is verified through simulation studies using SIMULINK models and experimental studies that use an up-down motion control apparatus with a flexible horizontal beam.

Section 2 briefly describes the idea of input shaping control. Section 3 defines the robustness function and the performance measure for residual vibration control and utilizes simulation studies to verify the definitions. Section 4 describes the design of the experimental apparatus and demonstrates the validity of the robustness function and performance measure by experiments. Section 5 concludes the paper.

2 Input Shaping Control

There are several ways to suppress unwanted residual vibrations. The most well-known technique is input shaping control. Figure 1 shows a block diagram for the input shaping control in which an input shaper convolves command input $r(t)$ with a sequence of impulses. For a single mode vibration system with an underdamped second-order linear dynamics, the output of the so-called zero vibration (ZV) input shaper, $r^*(t)$, is represented by

$$r^*(t) = r(t) [A_1 \delta(t-t_1) + A_2 \delta(t-t_2)] = \int_0^t r(\tau) \cdot [A_1 \delta(t-\tau-t_1) + A_2 \delta(t-\tau-t_2)] d\tau$$

where $\delta$ indicates the Dirac delta function. If $r(t)$ is given by a step function $c \cdot h(t)$, then

$$r^*(t) = c [A_1 h(t-t_1) + A_2 h(t-t_2)]$$

where $h(t)$ indicates the Heaviside step function and $c$ is a constant value. The parameters $t_1, t_2, A_1,$ and $A_2$ in Eq. (2) are obtained from constraint equations given by the ZV shaper [4–6,17].

$$t_1 = 0, \quad t_2 = \frac{\pi}{\omega_n \sqrt{1-\xi^2}}, \quad A_1 = \frac{e^{i\pi / \sqrt{1-\xi^2}}}{1 + e^{i\pi / \sqrt{1-\xi^2}}}, \quad A_2 = \frac{1}{1 + e^{i\pi / \sqrt{1-\xi^2}}}$$

where $t_1$ and $t_2$ are the time locations of impulses, $A_1$ and $A_2$ are the magnitudes of the impulses, and $\omega_n$ and $\xi$ are the natural frequency and damping ratio of the flexible structure, respectively. Note that $t_1 = 0$ and $A_1 + A_2 = 1$ without loss of generality. The duration of $t_2$ is one-half period of the damped vibration.

A zero vibration and derivative (ZVD) input shaper composed of three impulses is more robust to modeling errors than the ZV input shaper. This is achieved by adding additional constraints on the derivative of residual vibration magnitudes and is given by [4–6,17]

$$t_1 = 0, \quad t_2 = \frac{\pi}{\omega_n \sqrt{1-\xi^2}}, \quad t_3 = \frac{2\pi}{\omega_n \sqrt{1-\xi^2}}, \quad A_1 = \frac{e^{i\pi / \sqrt{1-\xi^2}}}{(1 + e^{i\pi / \sqrt{1-\xi^2}})^2}, \quad A_2 = \frac{2e^{i\pi / \sqrt{1-\xi^2}}}{(1 + e^{i\pi / \sqrt{1-\xi^2}})^2}$$

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where

\[ n = \frac{1}{1 - \xi^2} \]

is the final impulse time of the input shaper. As an example, a plot of function \( V \) in Eq. (12) with respect to natural frequency error is shown in Fig. 2, which is a display of the sensitivity curve. The ZVD input shaper is more robust than ZV input shaper in the conventional sense since ZVD curve is more flat than ZV curve near \( \omega = \omega_\text{modelled} / \omega_\text{actual} = 1 \). However, the final impulse time \( t_N \) in Eq. (12) varies according to the modeling error in \( \omega_n \) and so it is not reasonable to compare robustness performances of various residual vibration controls with this sensitivity curve, in spite of the fact that it represents a rough tendency of robustness of the input shaping control. A better way to compare the robustness of various control logics for residual vibration suppression is to compare the amount of overshoot and undershoot of each input shaping control during a specified time interval. By specifying defined time intervals in advance, the effect of the vibration-suppressing speed in the analysis can be considered.

In this paper, the actual natural frequency and the damping ratio are fixed, and the model natural frequency and damping ratio are varied for experimental convenience. Note that the sensitivity curve is conventionally plotted using varying \( \omega_\text{actual} \) for fixed \( \omega_\text{modelled} \) so that x-axis becomes \( \omega_\text{actual} / \omega_\text{modelled} \) instead of \( \omega_\text{actual} / \omega_\text{actual} \).

Before defining a performance measure for the robustness of residual vibration control, we define first a robustness function \( R \) as the ratio of residual vibration with control to that without control. The robustness function should be independent of the magnitude of reference input and also of the actuator dynamics that exists between an input shaper and a flexible system. Furthermore, the robustness function should include the effect of response speeds as well as amplitude sizes of residual vibrations. Plotting the robustness function with respect to modeling errors is called a robustness curve in a residual vibration control.

The robustness function with the above features is well defined by the ratio of the integral of squared errors (ISEs) of residual vibrations for the case with input shaper or any other residual vibration control logic to the one for the case without input shaper or any other residual vibration control logic during a prespecified time interval. The errors of residual vibrations are measured with respect to a nominal motion or a motion base. That is,

\[
R(\omega_\text{error}, \xi_\text{error}) = \frac{\text{ISE of input shaping case (} \omega_\text{actual} \times \xi_\text{actual}} {\text{ISE of no input shaping case}}
\]

\[
= \frac{\int_{t_i}^{t_f} (y_{IS} - y_{rigid})^2 \, dt} {\int_{t_i}^{t_f} (y_{noIS} - y_{rigid})^2 \, dt}
\]

(13)

where \( \omega_\text{error} \) and \( \xi_\text{error} \) represent modeling errors of natural frequency \( \omega_n \) and damping ratio \( \xi \), respectively. This definition is applied not only to the input shaping control but also to other residual vibration controls. The robustness function \( R \) is a function of the modeling errors \( \omega_\text{error} \) and \( \xi_\text{error} \) since residual vibration \( y_{IS} \) is a function of the modeling errors \( \omega_\text{error} \) and \( \xi_\text{error} \). In Eq. (13), \( y_{noIS} \) and \( y_{IS} \) are displacements of a flexible beam or cable without vibration-suppressing control logic (noIS implies no input shaping) and with vibration-suppressing control logic, respectively. The value of \( y_{rigid} \) is a displacement of a motion base on which the cable or the flexible beam is attached.
Figure 3 shows an example of vibratory motion of a flexible material attached to a motion base without residual vibration control. Rigid motion is represented as $y_{\text{rigid}}$ and flexible motion is represented as $y_{\text{flexible}}$. If the residual vibration is removed completely, then the value of $R$ is 0. If the residual vibration is the same with the case without residual vibration suppression control, then the value of $R$ is 1.

In the robustness function \( R \), the lower limit $t_i$ is appropriately selected according to residual vibration types. If residual vibration is of the same type, as shown in Fig. 3, in which motion command is smoothly rising instead of a step rise, $t_i$ may be selected as a value of the settling time of the rigid motion; in other words, 98% reaching time of the steady-state value. If residual vibration is of the same type, as shown in Fig. 4, in which the command is a step fashion, $t_i$ may be selected as final impulse time of the input shaper. The lower limit $t_i$ may differ depending on the residual vibration control logic that is used and is selected according to the time interval that under which the study is concerned. For example, $t_2$ or $t_3$ for the case in Fig. 4 may be selected as $t_i$, which is the second or third impulse instant of the ZV shaper or the ZVD shaper.

The final time $t_f$ may be selected as the lower limit $t_i$ plus a value of 3 times the period of the damped natural frequency, i.e., $t_f = t_i + 3 \cdot 2\pi/\omega_n$. In Fig. 4, the vibratory solid line is a step response of a second-order system with a natural frequency of 1 Hz and a damping ratio of 0.2. The dotted line represents the result of the ZV input shaping for the case without modeling errors. The dashdot line represents the result of the ZV input shaping for the case without modeling errors. Since the time interval of integration is independent of modeling errors, the suppressing speed of residual vibration is considered in the robustness function \( R \).

Figure 5 shows robustness curves by Eq. (13) for an underdamped second-order system with 1 Hz natural frequency and 0.2 damping ratio. Large values of $R$ for low values of $\omega_{\text{modeled}}/\omega_{\text{actual}}$ are due to big impulse times $t_2$ and $t_3$ by frequency modeling errors, as shown in Fig. 6. Note that $t_2$ or $t_3$ in the lower limit $t_i$ is a fixed value of the case without modeling errors and is independent of modeling errors. Actually, the result of Fig. 5(a) shows the well-known fact that the ZVD shaper has a penalty in settling time even if it is more robust than the ZV shaper in modeling errors, as shown in Fig. 6. Note that Fig. 2 does not show the settling time penalty. In contrast, Fig. 5(b) includes the effect of the settling time penalty. Note also that if necessary, the robustness curves in Fig. 5 may be plotted using varying $\omega_{\text{actual}}$ for fixed $\omega_{\text{modeled}}$ so that $x$-axis becomes $\omega_{\text{actual}}/\omega_{\text{modeled}}$ instead of $\omega_{\text{modeled}}/\omega_{\text{actual}}$.

We define a performance measure in the residual vibration control as the inverse number of the sum of the robustness function values at 70%, 80%, 90%, 100%, 110%, 120%, and 130% of the actual parameter value that is under investigation. Roughly speaking, it represents the total amount of residual vibrations within ±30% modeling errors in the model parameter that is concerned. It can be written in the following equation:
where \( x \) is \( \omega_n, \zeta \), or any other parameters that are under investigation for robustness analysis. PM takes a value between 0 and positive infinity. A larger value indicates better robustness performance.

Table 1 summarizes the simulation results. From this table, it can be concluded that the ZVD input shaping is better than the ZV input shaping in robustness to modeling errors only for the time intervals after \( t_3 \). If we consider the time interval from \( t_2 \) to \( t_f \), the robustness of the ZV input shaping is better than the robustness of the ZVD input shaping, contrary to what has been conventionally stated. This fact is supported by the robustness curve in Fig. 5(a) and the time response curves in Fig. 6, in which solid lines are input shaping results of no error case and dotted lines and dashdot lines are input shaping results of \( \pm 30\% \) modeling errors in natural frequency. Table 1 and Fig. 7 are the various input shaping results for the second-order system with the actual natural frequency \( \omega_n = 1 \) Hz and the damping ratio \( \zeta = 0.2 \). In Fig. 7, gray bars (left) are PM when \( t_i = t_2 \), and brown ones (right) are PM when \( t_i = t_3 \) for step responses of the second-order system.

From this analysis (Fig. 7), we can say that the ZVD input shaper is the best among the four input shapers studied in robustness if response speed is not an important issue. The ZV and the UM-ZV shapers are good in robustness if fast responses are required.

Table 1  Performance measures for various input shapers

<table>
<thead>
<tr>
<th>Shaper</th>
<th>( t_i = t_2 )</th>
<th>( t_i = t_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>ZV in ( \omega_n ) errors</td>
<td>0.82</td>
<td>0.78</td>
</tr>
<tr>
<td>ZV in ( \zeta ) errors</td>
<td>20.8</td>
<td>20.8</td>
</tr>
<tr>
<td>ZVD in ( \omega_n ) errors</td>
<td>0.26</td>
<td>1.84</td>
</tr>
<tr>
<td>ZVD in ( \zeta ) errors</td>
<td>0.43</td>
<td>1751</td>
</tr>
<tr>
<td>UM-ZV in ( \omega_n ) errors</td>
<td>0.75</td>
<td>0.74</td>
</tr>
<tr>
<td>UM-ZV in ( \zeta ) errors</td>
<td>19.0</td>
<td>15.6</td>
</tr>
<tr>
<td>EI in ( \omega_n ) errors</td>
<td>0.28</td>
<td>1.84</td>
</tr>
<tr>
<td>EI in ( \zeta ) errors</td>
<td>0.44</td>
<td>11.9</td>
</tr>
</tbody>
</table>

4 Experimental Verification

To experimentally show the validity of the proposed performance measure of residual vibration control, we designed an experimental apparatus with a 1DOF up-down motion. Figure 8 shows the experimental apparatus for suppressing residual vibrations, which includes an ac motor, a ball-screw, a servo driver, and...