

Theory of Computation

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Class Information

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Textbook: Introduction to the Theory of Computation

- Michael Sipser (MIT), 2nd Edition (2006) or 3rd Edition (2013)
- Cover to Ch.4 (p.214) and approximately 15 pages in every week.
- **Students are expected to study 6–9 hours in a week.**
- Course web page: <http://home.konkuk.ac.kr/~wnam/class/tc2017/>
- Grade – attendance: 10%, homework: 20%, midterm exam: 35%, final exam: 35%.

“The theory of computation” comprises the fundamental mathematical properties of computer hardware, software, and certain applications.

- We seek to determine what can and cannot be computed, how quickly, with how much memory, and on which type of computational model.
- Theoretical computer science does have many fascinating big ideas, but it also has many small and sometimes dull details that can be tiresome.
- My primary objective is to expose you to the exciting aspects of computer theory, without getting bogged down in the drudgery.
- Imitation game: <https://www.youtube.com/watch?v=nuPZUUED5uk>
- Alan Turing: https://en.wikipedia.org/wiki/Alan_Turing
- https://ko.wikipedia.org/wiki/%EC%95%A8%EB%9F%B0_%ED%8A%9C%EB%A7%81

Preface

Theory shows you a new, simpler, and more elegant side of computers.

- The best computer designs and applications are conceived with elegance in mind.
- A theoretical course can heighten your aesthetic sense and help you build more beautiful systems.

Theory is good for you because studying it expands your mind.

- Computer technology changes quickly.
- Specific technical knowledge, though useful today, becomes outdated in just a few years.
- Consider instead the abilities to think, to express yourself clearly and precisely, to solve problems, and to know when you haven't solved a problem.
- These abilities have lasting value.

Chapter 0. Introduction

0.1 Automata, Computability, and Complexity

What are the fundamental capabilities and limitations of computers?

- This question goes back to the 1930s when mathematical logicians first began to explore the meaning of computation.
- In each of the three areas—**automata**, **computability**, and **complexity**—this question is interpreted differently, and the answers vary according to the interpretation.

Complexity Theory

- Computer problems come in different varieties; some are easy, and some are hard.
- For example, the sorting problem is an easy one.
- Scheduling problem: If you have just a thousand classes, finding the best (**optimal**) schedule may require centuries, even with a supercomputer.
- What makes some problems computationally hard and others easy?

0.1 Automata, Computability, and Complexity

Computability Theory

- During the first half of the 20th century, mathematicians such as Kurt Gödel, Alan Turing, and Alonzo Church discovered that certain basic problems cannot be solved by computers.
- In complexity theory, the objective is to classify problems as easy ones and hard ones.
- In computability theory the classification of problems is by those that are solvable and those that are not.

Automata Theory

- Automata theory deals with the definitions and properties of mathematical models of computation.
- These models play a role in several applied areas of computer science.
 - One model, called the **finite automaton**, is used in text processing, compilers, and hardware design.
 - Another model, called the **context-free grammar**, is used in programming languages and artificial intelligence.

0.2 Mathematical Notions and Terminology

Sets

- **set:** $\{7, 21, 57\}$
- **elements** or **members:** $7 \in \{7, 21, 57\}, 8 \notin \{7, 21, 57\}$
- **subset** $A \subseteq B$, **proper subset** $A \subset B$
- **multiset:** $\{7\}$ and $\{7, 7\}$ are different as multisets but identical as sets.
- **infinite set:** $\{1, 2, 3, \dots\}$
- **empty set:** \emptyset
- **union** $A \cup B$, **intersection** $A \cap B$, **complement** \bar{A}
- **Venn diagram**

Sequences and Tuples

- A **sequence** of objects is a list of these objects in some order.
- $(7, 21, 57)$ is not the same as $(57, 7, 21)$.
- $(7, 7, 21, 57)$ is different from both of the other sequences, whereas the set $\{7, 21, 57\}$ is identical to the set $\{7, 7, 21, 57\}$.
- Finite sequences often are called **tuples**. A sequence with k elements is a **k -tuple**. Thus $(7, 21, 57)$ is a 3-tuple. A 2-tuple is also called an **ordered pair**.
- The **power set** of A is the set of all subsets of A .
- If A and B are two sets, the **Cartesian product** or **cross product** of A and B , written $A \times B$, is the set of all pairs wherein the first element is a member of A and the second element is a member of B .
- Example 0.5: If $A = \{1, 2\}$ and $B = \{x, y, z\}$,
 $A \times B = \{(1, x), (1, y), (1, z), (2, x), (2, y), (2, z)\}$.
- Example 0.6: If A and B are as in Example 0.5, $A \times B \times A = ?$

Functions and Relations

- A **function** takes an input and produces an output. $f(a) = b$
- A function also is called a **mapping**, and, if $f(a) = b$, we say that f maps a to b .
- The set of possible inputs to the function is called its **domain**.
- The outputs of a function come from a set called its **range**. $f : D \rightarrow R$.
- When the domain of a function f is $A_1 \times \cdots \times A_k$ for some sets A_1, \dots, A_k , the input to f is a k -tuple (a_1, a_2, \dots, a_k) and we call the a_i the **arguments** to f .
- A function with k arguments is called a **k -ary function**, and k is called the **arity** of the function.
- If k is 1, f has a single argument and f is called a **unary function**. If k is 2, f is a **binary function**.
- A **predicate** or **property** is a function whose range is $\{\text{TRUE}, \text{FALSE}\}$. e.g., $even()$.
- A property whose domain is a set of k -tuples $A_1 \times \cdots \times A_k$ is called a **relation**, a **k -ary relation**.

Functions and Relations

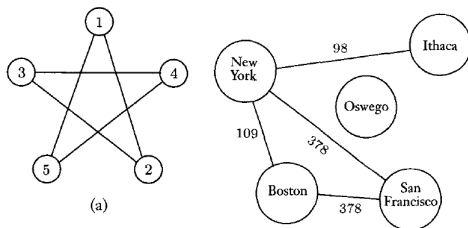
- Example 0.10 Scissors-Paper-Stone game
- In a children's game called Scissors-Paper-Stone, the two players simultaneously select a member of the set {SCISSORS, PAPER, STONE} and indicate their selections with hand signals.
- If the two selections are the same, the game starts over.
- If the selections differ, one player wins, according to **the relation beats**.

<i>beats</i>	SCISSORS	PAPER	STONE
SCISSORS	FALSE	TRUE	FALSE
PAPER	FALSE	FALSE	TRUE
STONE	TRUE	FALSE	FALSE

- Sometimes describing predicates (relations) with **sets** instead of functions is more convenient.
- The predicate $P : D \rightarrow \{\text{TRUE}, \text{FALSE}\}$ may be written (D, S) , where $S = \{a \in D \mid P(a) = \text{TRUE}\}$, or simply S if the domain D is obvious from the context.
- Hence the relation **beats** may be written $\{(\text{SCISSORS}, \text{PAPER}), (\text{PAPER}, \text{STONE}), (\text{STONE}, \text{SCISSORS})\}$.

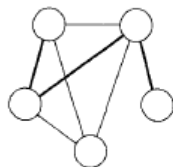
Graphs

- An **undirected graph**, or simply a **graph**, is a set of points with lines connecting some of the points.
- The points are called **nodes** or **vertices**, and the lines are called **edges**.
- The number of edges at a particular node is the **degree** of that node.
- In a graph G that contains nodes i and j , the pair (i, j) represents the edge that connects i and j .
- If V is the set of nodes of G and E is the set of edges, we say $G = (V, E)$.
- A formal description of the graph in Figure 0.12(a) is $(\{1, 2, 3, 4, 5\}, \{(1, 2), (2, 3), (3, 4), (4, 5), (5, 1)\})$.
- Sometimes, for convenience, we label the nodes and/or edges of a graph, which then is called a **labeled graph**.

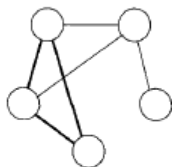


Graphs

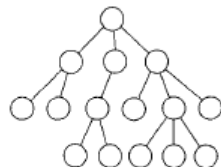
- We say that graph G is a **subgraph** of graph H if the nodes of G are a subset of the nodes of H , and the edges of G are the edges of H on the corresponding nodes.
- A **path** in a graph is a sequence of nodes connected by edges.
- A **simple path** is a path that doesn't repeat any nodes.
- A graph is **connected** if every two nodes have a path between them.
- A path is a **cycle** if it starts and ends in the same node.
- A **simple cycle** is one that contains at least three nodes and repeats only the first and last nodes.
- A graph is a **tree** if it is connected and has no simple cycles.



(a)



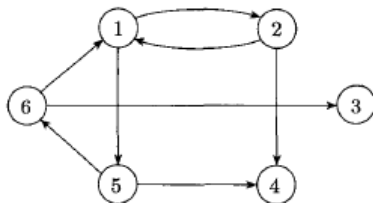
(b)



(c)

Graphs

- If it has arrows instead of lines, the graph is a **directed graph**.
- The number of arrows pointing from a particular node is the **outdegree** of that node, and the number of arrows pointing to a particular node is the **indegree**.
- The formal description of the graph in Figure 0.16 is $(\{1, 2, 3, 4, 5, 6\}, \{(1, 2), (1, 5), (2, 1), (2, 4), (5, 4), (5, 6), (6, 1), (6, 3)\})$.
- A path in which all the arrows point in the same direction as its steps is called a **directed path**.
- A directed graph is **strongly connected** if a directed path connects every two nodes.



Strings and Languages

- Strings of characters are fundamental building blocks in computer science.
- We define an **alphabet** to be any nonempty finite set.
- The members of the alphabet are the **symbols** of the alphabet.
- $\Sigma_1 = \{0, 1\}$,
 $\Sigma_2 = \{a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z\}$,
 $\Gamma = \{0, 1, x, y, z\}$
- A **string over an alphabet** is a finite sequence of symbols from that alphabet.
- If $\Sigma_1 = \{0, 1\}$, then 01001 is a string over Σ_1 .
- If $\Sigma_2 = \{a, b, c, \dots, z\}$, then abracadabra is a string over Σ_2 .
- If w is a string over Σ , the **length** of w , written $|w|$, is the number of symbols that it contains.
- The string of length zero is called the **empty string** and is written ε .

Strings and Languages

- If w has length n , we can write $w = w_1w_2 \cdots w_n$ where each $w_i \in \Sigma$.
- The reverse of w , written $w^{\mathcal{R}}$, is the string obtained by writing w in the opposite order (i.e., $w_nw_{n-1} \cdots w_1$).
- String z is a **substring** of w if z appears consecutively within w .
- If we have string x of length m and string y of length n , the **concatenation** of x and y , written xy , is the string obtained by appending y to the end of x .
- The **lexicographic ordering** of strings is the same as the familiar dictionary ordering.
- A **language** is a set of strings.

Boolean Logic

- **Boolean logic** is a mathematical system built around the two values TRUE and FALSE.
- The values TRUE and FALSE are called the **Boolean values** and are often represented by the values 1 and 0.
- **Boolean operations: negation**, NOT, \neg , **conjunction**, AND, \wedge , **disjunction**, OR, \vee .
- $P \wedge Q$: P and Q are called the **operands** of the operation.
- **exclusive or**, XOR, \oplus , **equality**, \leftrightarrow , **implication**, \rightarrow .

$$0 \oplus 0 = 0$$

$$0 \oplus 1 = 1$$

$$1 \oplus 0 = 1$$

$$1 \oplus 1 = 0$$

$$0 \leftrightarrow 0 = 1$$

$$0 \leftrightarrow 1 = 0$$

$$1 \leftrightarrow 0 = 0$$

$$1 \leftrightarrow 1 = 1$$

$$0 \rightarrow 0 = 1$$

$$0 \rightarrow 1 = 1$$

$$1 \rightarrow 0 = 0$$

$$1 \rightarrow 1 = 1$$